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Nuclear structure functions

D Indumathi*

Centre for Theoretical Studies, Indian Institute of Science, Bangalore-560 012, India

Abstract : We present a general review of currently popular models of bound nucleon structure functions. The Q^2 dependence predicted by various models is highlighted; in principle, this can be used to experimentally distinguish between various models.

Keywords : Nuclear structure functions, Q^2 dependence

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1. Introduction

Structure functions of bound and free nucleons are not equal : this is called the EMC effect [1]. Although this discovery was made nearly fifteen years ago, the origin of the EMC effect is still an open problem [2]. In deep inelastic scattering of leptons off a nucleus of mass A, the average nuclear structure function, $F_2^A(x, Q^2)$, was thought to be an incoherent sum :

$$F_2^A(x, Q^2) = \frac{1}{A}\left[ZF_2^p(x, Q^2) + (A-Z)F_2^n(x, Q^2)\right],$$

where the kinematic variables, $x = Q^2/(2p.q)$, $-q^2 = Q^2$ represent the Bjorken scaling variable and the momentum transfer from the lepton to the hadron of momentum p. Here $F_2^{p(n)}$ represents the proton (neutron) structure function respectively.

This assumption was made, because corrections due to nuclear binding (for a typical potential well depth of around 40 MeV) were expected to be about 1–4%. For nuclei with equal number of protons and neutrons, *i.e.*, $Z = A - Z = A/2$,

$$F_2^A(x, Q^2) = \frac{Z}{A}\left(F_2^p(x, Q^2) + F_2^n(x, Q^2)\right). \tag{1}$$

*Present Address : Mehta Research Institute, Allahabad-221 506, India

which is to be compared with the average free nucleon structure function,

$$F_2^D(x, Q^2) \equiv \frac{1}{2}\left(F_2^p(x, Q^2) + F_2^n(x, Q^2)\right). \tag{2}$$

Hence, at first glance, it appears as if the ratio of nuclear and free nucleon structure functions,

$$R^A \quad \frac{F_2^A}{F_2^D} = 1, \tag{3}$$

for all x, Q^2. Nuclear targets were therefore used to improve the statistics in the experiment, since the total cross section is proportional to A. It was expected that there would be deviations from this value, at very small and very large x values, due to nuclear shadowing and Fermi motion respectively. However, when the first data was taken by the EMC in 1982 [1], it was seen that R^A was, in general, not equal to 1 (see Figure 1). An attempt to explain this phenomenon led to the development of various models of nuclear structure

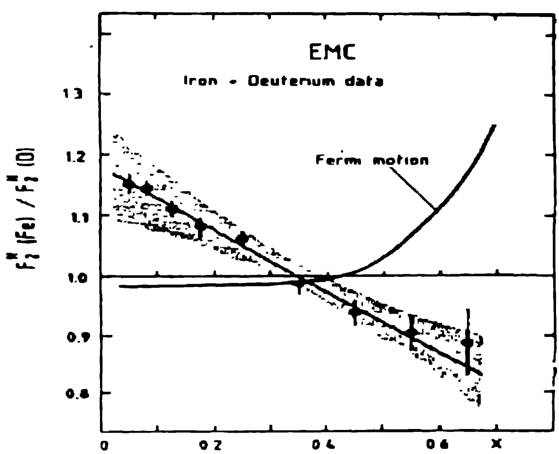


Figure 1. The ratios of the bound and free nucleon structure function as first determined by the EMC Collaboration [1] The solid curve shows the theoretical expectation at that time.

functions. All of them have various predictions for R^A, the latest data for which come from the NMC and E665 collaborations [3,4] for the nuclei, He, Li, C, Ca, Sn, *etc*. (See Figures 4, 6 and 7 for the data for some of these nuclei). It is seen that R^A is typically smaller than one for small x, $x < 0.05$, and for very large x, $x > 0.3$, and larger than one for intermediate values of x. The small- and intermediate-x regions are usually called the shadowing and the antishadowing regimes.

Data also exists for the Drell Yan ratio in $p\,A$ collisions from the E772 collaboration [5]. This indicates shadowing of the sea quarks, but no antishadowing. Information on the nuclear gluon distribution is available from J/ψ production in both μA and pA collisions; however, the results are fairly controversial and we shall not discuss them further here.

There are many models that describe the modification of the parton distributions inside a bound nucleon. Each model is based on different phenomena and applies in different kinematic ranges. Due to lack of time, we will discuss here only some models (typically representative of a class of similar models). A list of models and their region of applicability is neatly represented in the schematic shown in Figure 2, taken from Ref. [2];

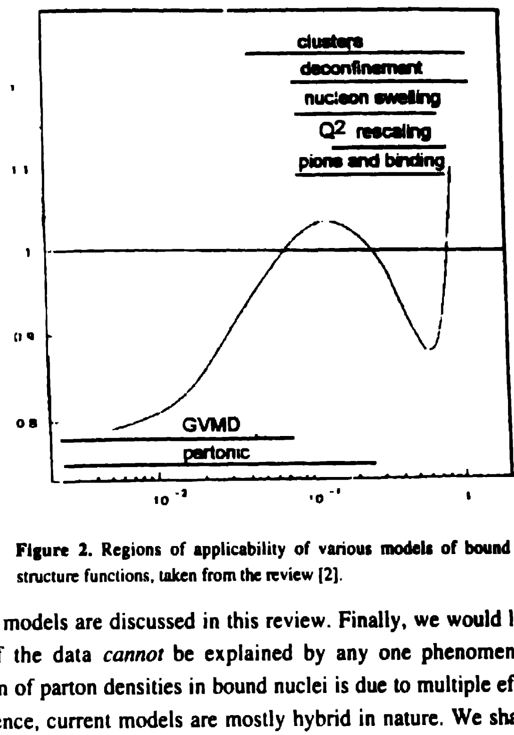


Figure 2. Regions of applicability of various models of bound nucleon structure functions, taken from the review [2].

many more models are discussed in this review. Finally, we would like to emphasise that all parts of the data *cannot* be explained by any one phenomenon. We believe that modification of parton densities in bound nuclei is due to multiple effects occurring in the nucleus. Hence, current models are mostly hybrid in nature. We shall concentrate on the small and intermediate x regions in our discussions, ignoring Fermi motion effects at very large x values. We begin by discussing the rescaling model, which was chronologically one of the earliest models to explain the "traditional" (large-x depletion) EMC effect.

2. Rescaling models

These use nuclear binding to explain the modification of nuclear parton densities. The rescaling can be either in x [6] or Q^2 [7,8]. Their characteristic feature is an increase in the confinement size in a bound nucleon, $\lambda_A > \lambda_N$. Hence, Q^2 in a bound nucleon is effectively increased by an amount,

$$Q^2 \to \xi Q^2 \,;\, \xi(Q^2) = \left(\frac{\mu_N^2}{\mu_A^2}\right)^{\alpha_s(\mu_A^2)/\alpha_s(Q^2)}, \tag{4}$$

or, equivalently, x increases by a factor $1 + \bar{\varepsilon}/M_N$, where $\bar{\varepsilon}$ is the average one-nucleon separation energy, and M_N the mass of the nucleon. This results in a decrease of R^A at large x but cannot explain the small x shadowing. There is no "explanation" for the change of scale; the model only provides a framework for discussing it. Furthermore, it is not clear whether the sea densities are depleted as well or just the valence densities.

3. Parton fusion models

These were initially discussed prior to the availability of any data [9]. However, the models have undergone many modifications in detail. The underlying idea is that a parton with momentum fraction x of a parent hadron with momentum p, is localised to within $\Delta z \sim 1/(xp)$, from the uncertainty principle. On the other hand, the average internucleon separation (in the Breit frame) is $\Delta z_N \sim 2R_N M_N/p$, where R_N is the nucleon radius. When $\Delta z \sim \Delta z_N$, partons of different nucleons start to overlap spatially. This happens when

$$x < x_N = 1/(2R_N M_N) \sim 0.1; \tag{5}$$

the effect saturates when

$$x < x_A = 1/(2R_N m_N) = n_N A^{-1/3}, \tag{6}$$

where R_A, the nuclear radius, is not to be confused with the ratio of structure functions or densities, R^A. The idea is that overlapping partons can interact and fuse, and so (a) reduce the parton density at small $x < 0.1$, and (b) correspondingly increase it at intermediate x. Hence the ratio of bound to free nucleon structure functions is parametrised as

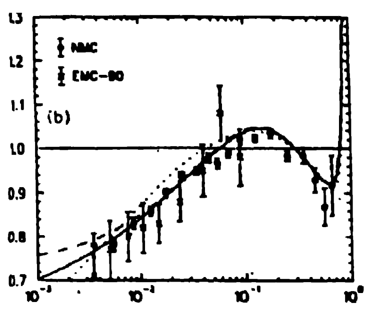
$$\begin{aligned} R^A &= 1\,; && x_N < x < 1\,; \\ &= 1 - K\left(\frac{x_N}{x} - 1\right); && x_A < x < x_N\,; \\ &= 1 - K\left(\frac{x_N}{x_A} - 1\right); && x < x_A. \end{aligned} \tag{7}$$

Here K is an unknown, free parameter and $(x_N/x - 1)$ is the number of overlapped nucleons. This was more of a geometric counting approach, and did not discuss the origin of shadowing, *i.e.*, the mechanism of fusion. That is, the K factor was fitted to data. Soon a QCD-based purely perturbative calculation appeared [10,11]. The usual DGLAP evolution equations [12] for free nucleon densities are linear in the densities. The GLR-based Müller-Qiu equations are non-linear. The nonlinear terms arise when the overlap of partons (or an increase in density) allows two gluons or a quark-antiquark pair to fuse to one gluon, in a process which is like the inverse of the usual parton "splitting" diagrams. The resulting evolution equations for quarks and gluons appear as follows :

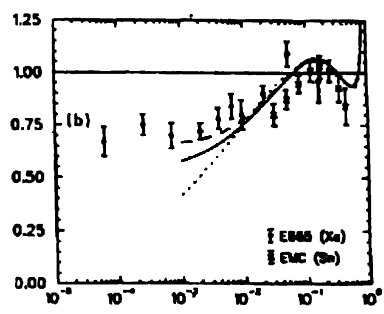
$$\frac{\partial q}{\partial \ln Q^2}(x, Q^2) = \frac{\alpha_s}{2\pi}\left[P_{qq} \otimes q + P_{qg} \otimes g\right] - \frac{27\alpha_s^2}{160 Q^2 R_N^2}\,\theta(x_N - x)(xg)^2 + \text{HDT};$$

$$\frac{\partial g}{\partial \ln Q^2}(x, Q^2) = \frac{\alpha_s}{2\pi}\left[P_{gq} \otimes q + P_{gg} \otimes g\right] - \frac{81\alpha_s^2}{16 Q^2 R_N^2}\,\theta(x_N - x)\int_x^{x_N}\frac{dz}{z}(zg)^2 \tag{8}$$

The first term on the RHS corresponds to the usual DGLAP term and HDT refers to higher dimensional gluon terms [11]. Note that the extra terms due to fusion come with a relative negative sign, and so deplete the densities at a given x (the equations are valid for small $x < x_N$). The effect of quark-gluon fusion is rather small, i.e, quark shadowing is indirectly



Comparisons with EMC and NMC [8] data for Ca. $\xi_4^r = 1.86$.



Comparisons with E665 data for Xe [9] and EMC data for Sn [29]. Calculated results are for Xe. $\xi_4^r = 2.24$.

Figure 3. The ratio, R^A, according to the model [13] in comparison with data for Ca, Xe and Sn.

driven by the more dominant gluon-gluon fusion. Finally, these extra terms are associated with a $1/Q^2$ factor so that the depletion at small x must decrease or even disappear with increasing Q^2. The observed quantity is the ratio of the bound to free nucleon structure functions The free nucleon structure functions do not have any modification of $1/Q^2$ nature, but only the usual log Q^2 behaviour. (The probability of parton fusion is considered to be much smaller within a single nucleon). The bound nucleon structure functions, according to the above model, have a leading logQ^2 behaviour with a depletion term at small x which has a $1/Q^2$ behaviour. Hence, the small-x shadowing, although predicted to decrease with increasing Q^2 will vanish at a rate in between that of a logQ^2 and a $1/Q^2$ behaviour, according to this model. This model is in fact one of the most popular models to explain small-x shadowing behaviour in nuclei. The predictions of the hybrid model of Kumano and Miyama [13], which combines the ideas of rescaling and parton fusion, is shown in Figure 3, in comparison with available data for various nuclei. The fits are good;

however, the model predictions are extremely sensitive to the initial Q_0^2 from which the densities are evolved.

4. Vector meson dominance models

This class of models also attempts mainly to explain the small x shadowing. Here the basic idea [14] is that the interacting (virtual) photon fluctuates into a quark-antiquark pair, or, equivalently, a meson, which then interacts with the target proton or nucleus. Hence $\gamma^* p(A)$ scattering can be viewed as hadron-hadron scattering, with the photon propagator being expressed as

$$\gamma^* \text{ propagator} \sim \frac{1}{\left(Q^2 + M_v^2\right)^2}; \quad V = \rho, \omega, \phi, \ldots \tag{9}$$

The vector meson-nucleus cross section is obtained by Glauber multiple scattering; every scattering turns out to have an amplitude *opposite* in phase to the previous one, and of decreasing magnitude :

$$M \sim A_0 - A_1 + A_2 + \cdots \sim A_0[1 - (\alpha)],$$

where $\alpha < 1$, thus leading to shadowing. Hence, at low x, the extra contribution to the nuclear structure function is [14]

$$\delta^V F_2^A(x, Q^2) = \frac{1}{A}\frac{Q^2}{\pi}\sum_V \frac{M_V^4}{\left(Q^2 + M_v^2\right)^2}\frac{\delta\sigma_{VA}}{f_V^2}. \tag{9a}$$

The model is again valid only at low x and cannot explain the conventional EMC effect. It not only predicts a significant decrease of shadowing with Q^2, but also predicts that shadowing decreases linearly as $1/Q^2$, disappearing totally by about $Q^2 \sim 10$ GeV^2. This may not be borne out by Drell Yan data [5]. The model predictions at low x for various nuclei are compared with data in Figure 4.

5. Nuclear effects and the parton model

This class of models [15] continues to use the linear DGLAP perturbative evolution equations with no fusion terms. Shadowing is then obtained by appealing to nuclear binding. Since bound nucleons lose typically an amount b (= the binding energy per nucleon ≤ 15 MeV) due to binding, bound nucleons have a larger spatial extent than free nucleons [7]. If δ_A is the relative increase in radius of a bound nucleon compared to a free one, due to the uncertainty principle, the momentum distribution (x distribution) of bound nucleons is different from free ones. However, at the starting low Q^2 scale from where the parton densities are evolved, the *number density* of partons as well as the *total momentum* carried by each type, remains conserved. These three constraints are sufficient to fix the bound nucleon densities in terms of the free parton distributions [15] and δ_A, which is a free

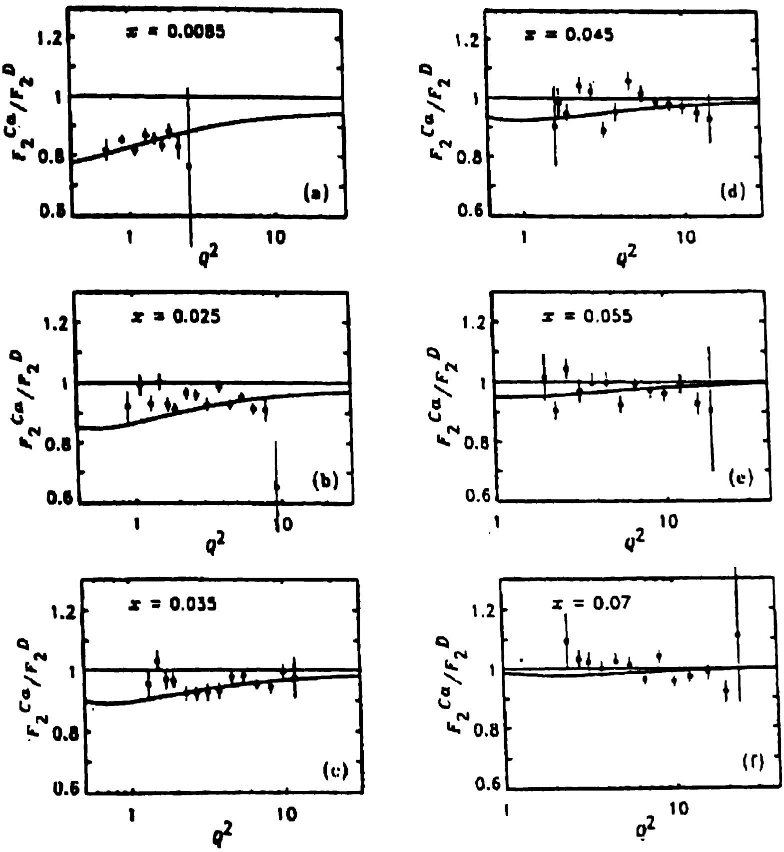


Figure 4. The Q^2 dependence of the ratio, R^A, at different values of x, according to the model [14] in comparison with NMC data for Ca [3].

parameter in the model. The result of this modification is a "pinching" of the x distribution, as shown in Figure 5.

The binding energy, b, corresponds to loss of energy of the bound nucleon, it is assumed that this energy loss is taken from the "mesonic" component or the sea quarks of the nucleons in this model. The bound nucleon sea density is thus reduced from the free nucleon one, $S_N(x, \mu^2)$, to

$$S_A(x,\mu^2) \doteq \left(1 - \frac{2b}{M_N\langle S_N(\mu^2)\rangle_2}\right) S_N(x,\mu^2), \qquad (9b)$$

where $\langle S_N \rangle_2$ is the momentum fraction carried by the sea in a free nucleon at the input scale. Since the mesons are soft, this is a small-x effect.

Hence, swelling prescribes the bound-nucleon densities at the input scale, μ^2. The sea densities are additionally depleted due to binding effects. These distributions are then evolved to any scale, $Q^2 > \mu^2$, using the DGLAP equations.

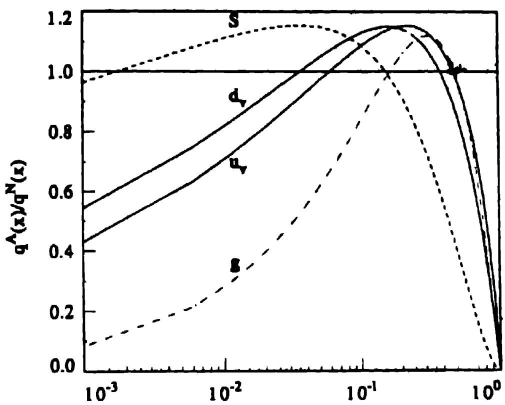


Figure 5. The effect of nucleon swelling on the calcium input distributions [15] · the ratios of the modified to unmodified densities are shown for the valence (u_v, d_v), sea (S) and gluon (g) densities with respect to the GRV [16] distributions for the free nucleon, and $\delta_A = 0.1$.

There is a further depletion of the sea densities which occurs at the time of scattering, due to nucleon nucleon interaction, arising from parton-nucleon overlap. As discussed in the parton fusion models, whenever the struck parton has a small enough momentum $x < x_N$, its wave function can overlap neighbouring nucleons. The subsequent interaction due to the overlap was seen to deplete the small x distributions by an amount K (see eq. (7)), where K was not calculable. Here, K is computed by analogy with binding. Let the energy loss due to overlap of sea quarks with one other nucleon be

$$U_s(Q^2) = \beta M_N \int_0^{x_N} x S_A(x, Q^2) \simeq \beta M_N \langle S_A(Q^2) \rangle_2,$$

and assume that the strength of this interaction is the same as that due to binding, *viz.*,

$$\beta = \frac{U_s(Q^2)}{M_N \langle S_A(Q^2) \rangle_2} = \frac{U(\mu^2)}{M_N \langle S_N(\mu^2) \rangle_2} \tag{10}$$

$U(\mu^2)$ being the binding energy between each pair of nucleons. Here, the possible Q^2 dependence of β is ignored. The only rôle of Q^2 here is to provide the impulse which allows the parton-nucleon overlap to occur. Then the extent of depletion of the sea at the scale Q^2

due to this overlap (called second binding effect) is given by eq. (7) with $K = 2\beta$. The model predictions for the x, Q^2, and A dependences of the ratios for He/D, C/D and Ca/D are shown in Figure 6.

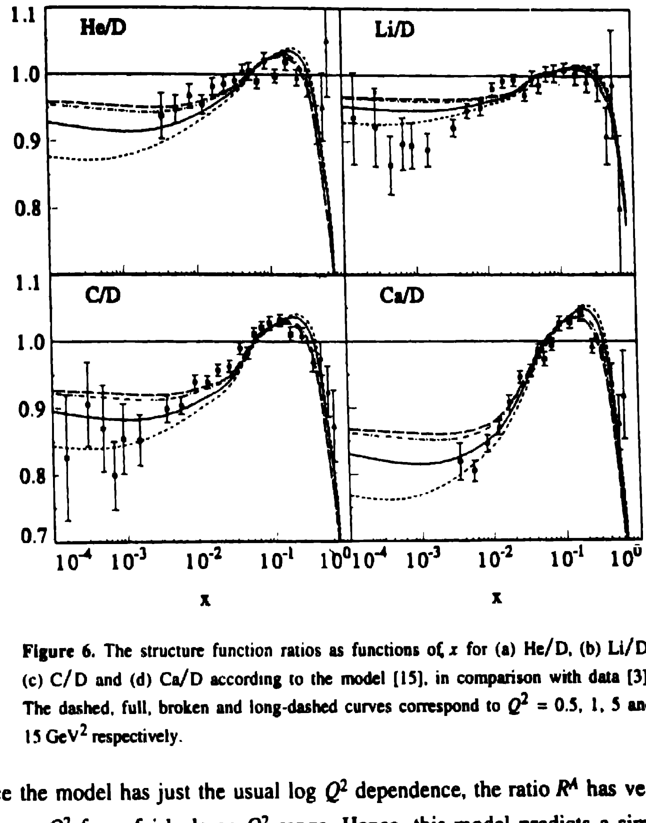


Figure 6. The structure function ratios as functions of x for (a) He/D, (b) Li/D, (c) C/D and (d) Ca/D according to the model [15], in comparison with data [3]. The dashed, full, broken and long-dashed curves correspond to Q^2 = 0.5, 1, 5 and 15 GeV^2 respectively.

Since the model has just the usual log Q^2 dependence, the ratio R^A has very little dependence on Q^2 for a fairly large Q^2 range. Hence, this model predicts a similar Q^2 behaviour for both bound and free nucleon distributions. Earlier data typically was consistent with little or no Q^2 dependence. Recent data on Sn/C from the EMC collaboration [18] seems to show a significant Q^2 dependence. This is the only data for which detailed Q^2 dependences are available, with very high statistics, and consequently small errorbars. This model is so far compatible with the data [17] as shown in Figure 7. However, continued evidence for a significant Q^2 dependence, especially at low x, will indicate that the Q^2 dependence of free and bound nucleon structure functions is *not* the same.

We add, in brief, that the model can be straightforwardly extended to the spin dependent case. Results [15,19] confirm that the ratio of the spin dependent bound and free structure functions is similar to the unpolarised ratio, R^A. This has positive implications [19] for the extraction of spin dependent structure functions from lepton-nucleus polarised deep inelastic scattering experiments.

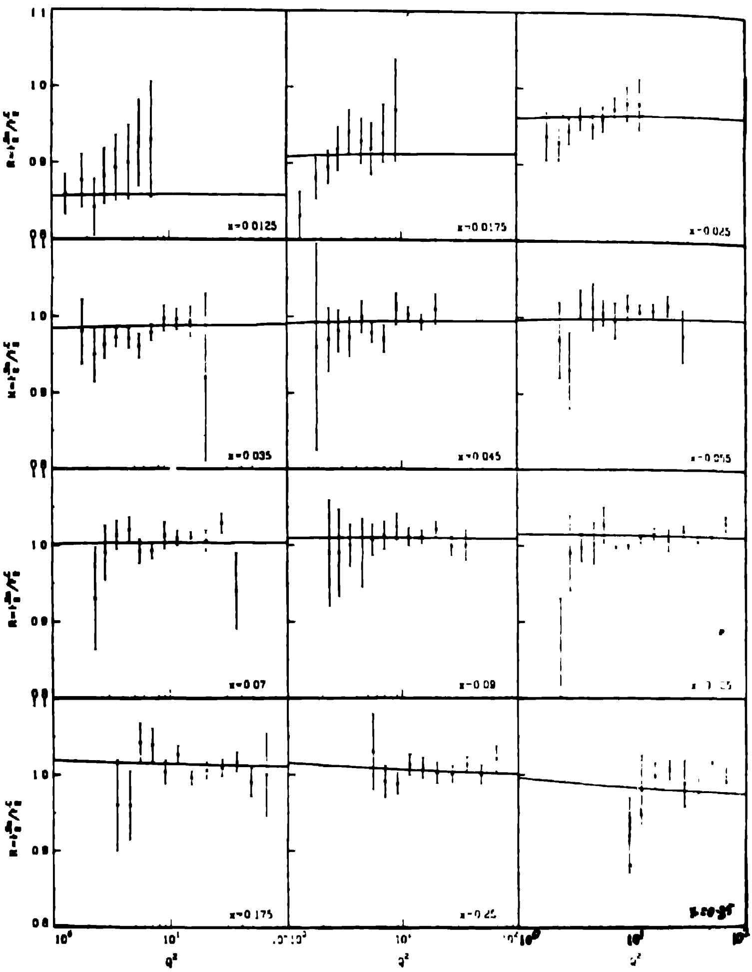


Figure 7. The model prediction [17] for the Q^2 dependence of the structure function ratio for Sn/C, in comparison with data from the NMC [18], with statistical and systematic errors added i. quadrature. Average (central bin) values of x are shown.

6. Summary and comments

We see that most models can fit the bound nucleon structure function, F_2^A (or, equivalently, the ratio, R^A), as a function of x over most of the x range over which data is available. However, these models generally differ with respect to the Q^2 dependence, especially at small x. This may be used to discriminate between them when more data becomes available at small x, over a substantial Q^2 range. This will establish if higher twist terms are significant, and enable the estimation of the bound nucleon gluon density, $g^A(x, Q^2)$, from $\partial F_2^A/\partial \ln Q^2$, about which very little is currently known.

F_2^A and Drell Yan data are complementary in nature. Hence, we cannot cross check the two sets of measurements against each other or establish the validity of any given model. Semi-inclusive π, K,... hadron production in deep inelastic lepton-nucleus scattering experiments can yield information on the valence combination, $(u_V + d_V)$, inside a nucleus, at all x, by measurements of suitable combinations of cross sections [20]. Such measurements can, in principle, discriminate between swelling and rescaling models.

Recently, uncertainty in AB collisions has been recognised to be due to nuclear absorption effects [21]. It may be possible to separate these from conventional (initial state) nuclear effects, provided the latter are well understood.

Many technical advances have recently occurred in the field of nuclear structure functions. This gives hope that "parametrisations" of bound nucleon parton distributions will soon be available, comparable in accuracy with free nucleon ones (like GRV [16], MRS [22], CTEQ [23], *etc*). This is important in the light of recent interest in the knowledge of bound nucleon parton densities, not as a tool in understanding nuclear/binding forces, but in order to be able to make suitable corrections to heavy ion collision cross sections, in the ongoing search for Quark Gluon Plasma [24].

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